Algebra 2/Trig
Unit 10 Notes Packet

#
Name:
Date:
Period:

Trigonometric Ratios and Functions

(1) Worksheet (Pythagorean Theorem and Special Right Triangles)

(2) Worksheet (Special Right Triangles)

(3) Page 772 - 773 #5 - 39 Column, #43, 44, and 45

(4) Page 772 - 774 #7 - 40 Column, #46 – 50

(5) Page 780 - 781 #25 – 59 odd

(6) Page 780 – 781 #26 – 58 even

(7) Worksheet (Unit Circle)

(8) Worksheet (Unit Circle)

(9) Page 803 #4 – 15

(10) Page 803 - 805 #16 - 35 Column, #56 and 60

(11) Page 803 - 805 #17 - 36 Column, #57 - 59, 64 and 65

(12) Page 810 #1 - 14

(13) Page 810 - 811 #15 - 36 Column, #50, 51

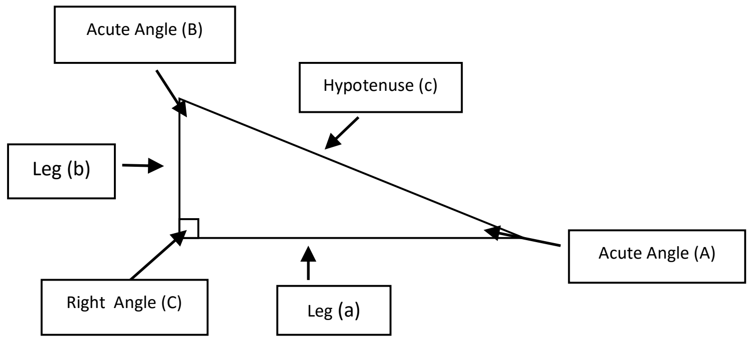
(14) Page 810 - 812 #17 - 37 Column, #52, 53

(15) Chapter Review

Trigonometry:

The study of triangular measure

Vocabulary



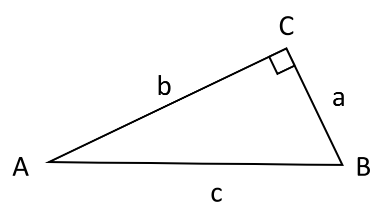
Right Triangle - A triangle that has one right angle and two acute angles
Right Angle - An angle that is 90 degrees (often represented with the letter C)
Acute Angle - An angle that is less than 90 degrees (often represented with letters A and B)
Hypotenuse - The side of a right triangle that is across from (opposite) the right angle (often represented with the letter c)
Leg - The side of a right triangle that is across from (opposite) the acute angle (often represented with the letters a and b)

Pythagorean Theorem Review

Directions: Find the missing side of the right triangle by using the Pythagorean Theorem

Pythagorean Theorem

$(\text{leg})^2 + (\text{leg})^2 = (\text{hypotenuse})^2$ or $a^2 + b^2 = c^2$



E1.) $a = 3$, $b = 4$ and $c = ?$

P1.) $a = 5$, $b = ?$ and $c = 13$

E2.) $a = ?$, $b = 36$ and $c = 39$

P2.) $a = 7$, $b = \sqrt{3}$ and $c = ?$

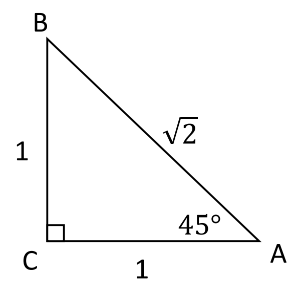
E3.) $a = 2\sqrt{5}$, $b = ?$ and $c = 12$

P3.) $a = ?$, $b = 3$ and $c = 3\sqrt{2}$

Special Right Triangles Review

There are two types of special right triangles (45-45-90) and (30-60-90) as represented by their angle measures. These triangles are special because their sides have a special ratio and therefore side measures can be found w/out the Pythagorean theorem or trigonometry equations.

45:45:90 is 1:1:$\sqrt{2}$



Directions: Find the missing side of the right triangle by using the 45:45:90 side ratios

E4.) a = 5, b = ? and c = ?

E5.) a = ? , b = ? and c = $9\sqrt{2}$

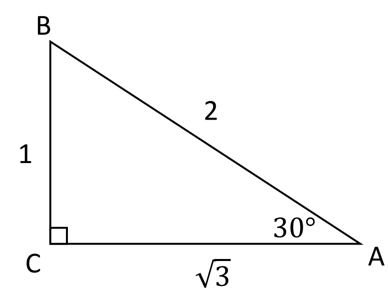
E6.) a =?, b = $2\sqrt{6}$ and c = ?

P4.) a = ?, b = 7 and c = ?

P5.) a = $7\sqrt{3}$, b = ? and c = ?

P6.) a = ?, b = ? and c = $10\sqrt{14}$

30:60:90 is 1:$\sqrt{3}$:2



Directions: Find the missing side of the right triangle by using the 30:60:90 side ratios

E7.) a = 5, b = ? and c = ?

E8.) a = ? , b = ? and c = 16

E9.) a =?, b = 10 and c = ?

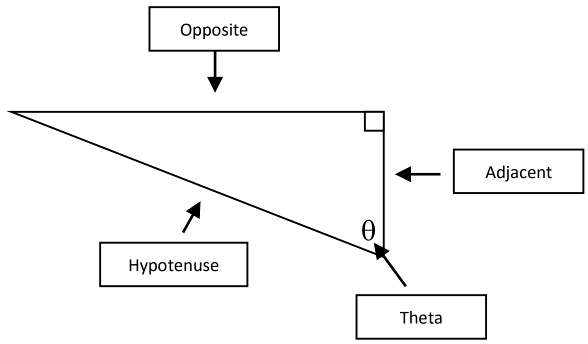
P7.) a = ?, b = $7\sqrt{3}$ and c = ?

P8.) a = $7\sqrt{3}$, b = ? and c = ?

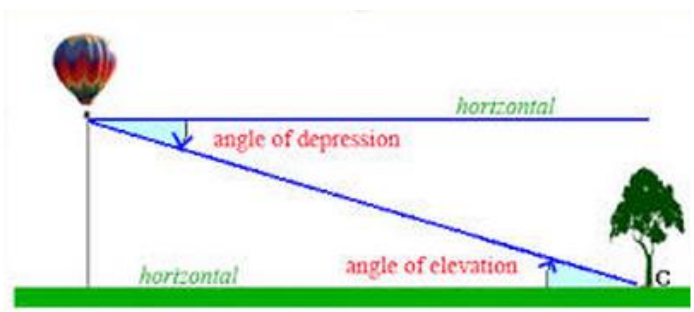
P9.) a = ?, b = ? and c = 10

Evaluating Trigonometric Functions

Vocabulary



Theta (θ) - A variable used in trigonometry to represent the measure of an acute angle
Adjacent Side - The side of the right triangle that is next to angle theta but not the hypotenuse
Opposite Side - The side of the right triangle that is across from theta



Angle of Elevation = Angle of Depression

Solving Trigonometric Equations

E1) Solve: $cos32° = \frac{x}{5}$ E2) Solve: $sin42° = \frac{13}{x}$ E3) Solve: $tan\theta = \frac{12}{13}$

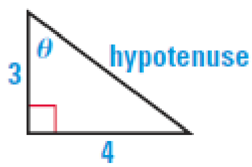
P1) Solve: $sin73° = \frac{x}{6}$ P2) Solve: $tan17° = \frac{13}{x}$ P3) Solve: $cos\theta = \frac{5}{17}$

$S_H^O C_H^A T_A^O$

Sin θ = $\frac{opposite}{hypotenuse}$ Cos θ = $\frac{adjacent}{hypotenuse}$ Tan θ = $\frac{opposite}{adjacent}$

Evaluate the trigonometric functions of the angle θ shown in the right triangle.

(E1.)

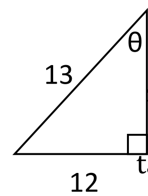


$\sin\theta =$

$\cos\theta =$

$\tan\theta =$

(P1.)

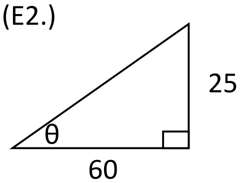


$\sin\theta =$

$\cos\theta =$

$\tan\theta =$

(E2.)

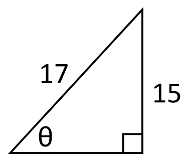


$\sin\theta =$

$\cos\theta =$

$\tan\theta =$

(P2.)



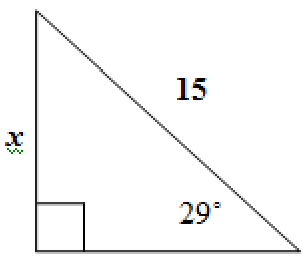
$\sin\theta =$

$\cos\theta =$

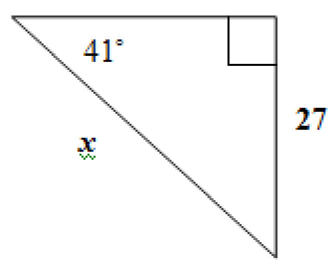
$\tan\theta =$

Solve for the missing side (x)

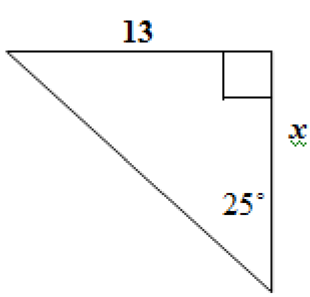
(E3.)



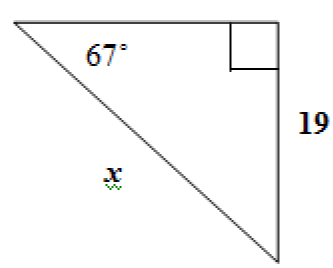
(P3.)



(E4.)

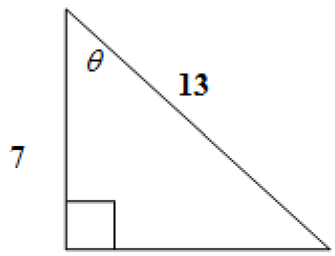


(P4.)

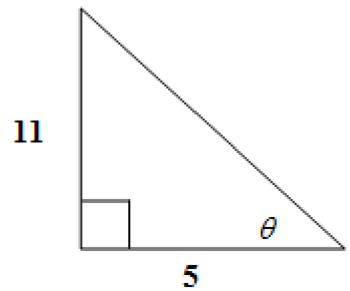


Solve for the missing angle (θ)

(E5.)

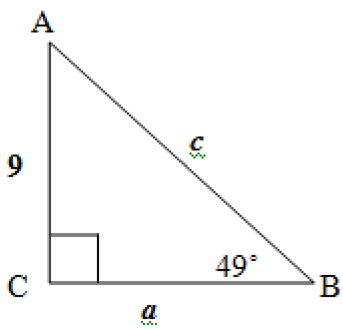


(P5.)

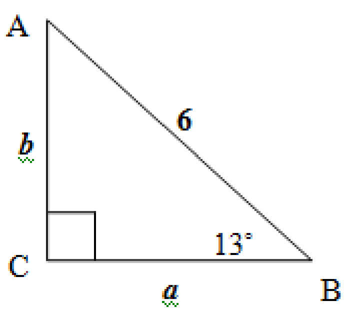


Solve the triangle. Find all missing sides and angles. Round sides to the nearest 10th and angles to the nearest minute.

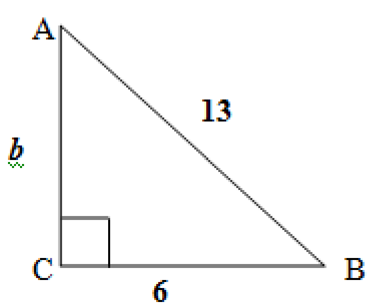
(E6.)



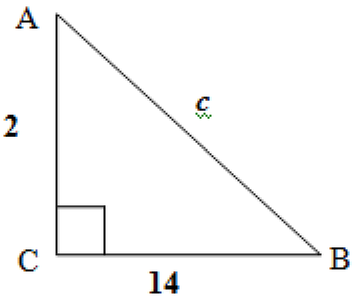
(P6.)



(E7.)



(P7.)



(E8.) Find a, if $B = 63^\circ$ and $c = 10$ feet

(P8.) Find c, if $B = 72^\circ$ and $a = 494$ miles

(E9.) A tower casts a shadow that is 60 feet long when the angle of elevation of the sun is 65°. How tall is the pole?

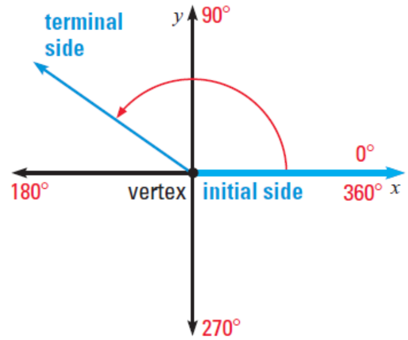
(P9.) You are standing 50 meters from a hot air balloon that is preparing to take off. The angle of elevation to the top of the balloon is 28°. Find the height of the balloon.

(E10.) A man is in a boat that is floating 175 feet from the base of a 200 - foot cliff. What is the angle of depression between the cliff and the boat?

(P10.) An airplane is directly above a beacon that is 10,000 feet from an airport control tower. The angle of depression from the plane to the base of the control tower is 6°. How high above the beacon is the plane?

Recall that an angle is formed by two rays that have a common endpoint, called the vertex. You can generate any angle by fixing one ray, called the ________________ side, and rotating the other ray, called the _________________ side, about the vertex.

In a coordinate plane, an angle whose vertex is at the origin and whose initial side is the positive *x*-axis is in ___________________ position.



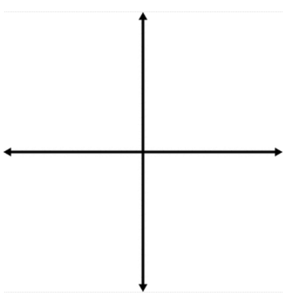
The measure of an angle is determined by the amount and direction of rotation from the initial side to the terminal side. The angle measure is _________________ if the rotation is counterclockwise, and ____________________ if the rotation is clockwise. The terminal side of an angle can make more than one complete rotation.

Two angles in standard position are called ____________________ angles if their terminal sides coincide. Coterminal angles can be found by adding or subtracting multiples of 360^0.

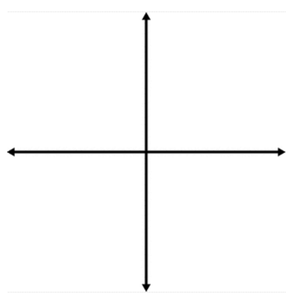
If the terminal side of an angle lies on an axis, then the angle is a ___________________ angle.

E1. Draw an angle with the given measure in standard position. Then tell in which quadrant the terminal side lies.

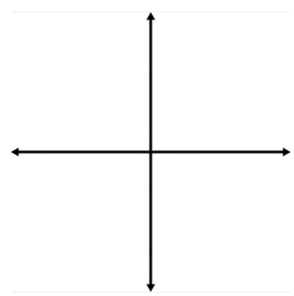
a. 210°



b. -45°

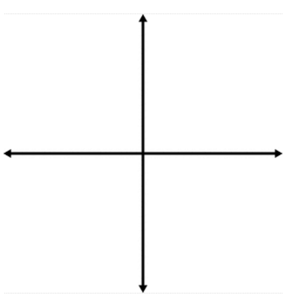


c. 510°

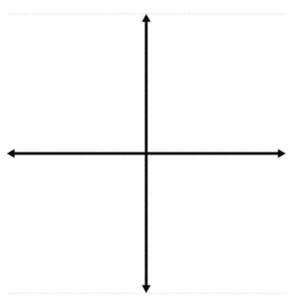


P1. Draw an angle with the given measure in standard position. Then tell in which quadrant the terminal side lies.

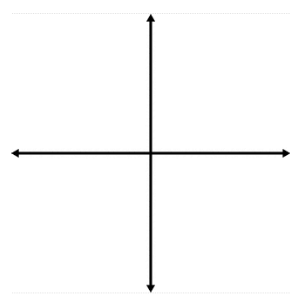
a. -120°



b. 400°



c. 135°



E2. Find one positive angle and one negative angle that are coterminal with (**a**) -60° and (**b**) 495°.

P2. Find one positive angle and one negative angle that are coterminal with (**a**) -100° and (**b**) 575°.

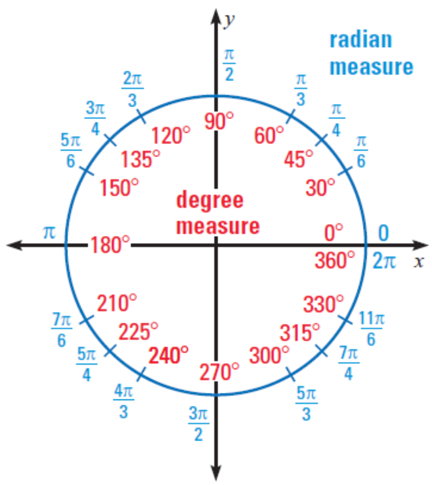
Radians and Degrees:
Angles can be measured in both radians and degrees (DD or DMS). To convert between degrees and radians it is important to know that a circle is 2π radians. Therefore 2π radians is equal to 360^0.

Convert From Radians to Degrees

- Multiply by

Convert From Degrees to Radians

- Multiply by



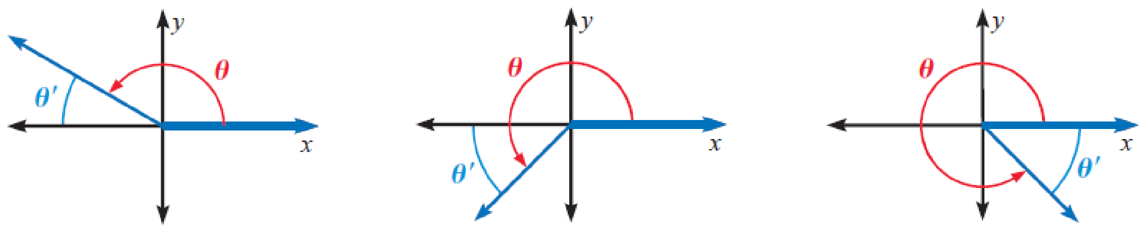
E3. **a.** Convert 110° to radians. **b.** Convert $-\frac{\pi}{9}$ radians to degrees.

P3. **a.** Convert 320° to radians. **b.** Convert $-\frac{5\pi}{12}$ radians to degrees.

13.3 Trigonometric Functions of Any Angle (I/2)

ANALYTIC TRIGONOMETRY is an extension of right triangle trigonometry. It takes place on the *x-y* plane. For, trigonometry as it is actually used in calculus and physics, is not about solving triangles. It becomes the mathematical description of things that rotate or vibrate, such as light, sound, the paths of planets about the sun or satellites about the earth. It is necessary therefore to have angles of any size, and to extend to them the meanings of the trigonometric functions.

The values of trigonometric functions of angles greater than 90^0 (or less than 0^0) can be found using corresponding acute angles called reference angles. A _________________ angle is formed by the terminal side of any non-quadrantal angle in standard position and the x – axis.



E1. Find the reference angle θ' for each angle θ.

a. $\theta = 130°$

b. $\theta = -\frac{3\pi}{4}$

c. $\theta = -250°$

P1. Find the reference angle θ' for each angle θ.

a. $\theta = 320°$

b. $\theta = -\frac{5\pi}{6}$

c. $\theta = \frac{8\pi}{3}$

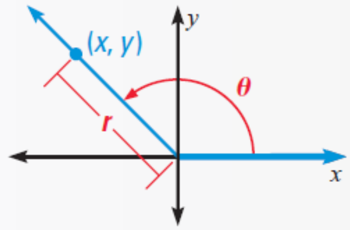
GENERAL DEFINITION OF TRIGONOMETRIC FUNCTIONS

Let θ be an angle in standard position and (x, y) be any point (except the origin) on the terminal side of θ. The six trigonometric functions of θ are defined as follows.

$$\sin\theta = \frac{y}{r} \qquad \csc\theta = \frac{r}{y},\ y \neq 0$$

$$\cos\theta = \frac{x}{r} \qquad \sec\theta = \frac{r}{x},\ x \neq 0$$

$$\tan\theta = \frac{y}{x},\ x \neq 0 \qquad \cot\theta = \frac{x}{y},\ y \neq 0$$

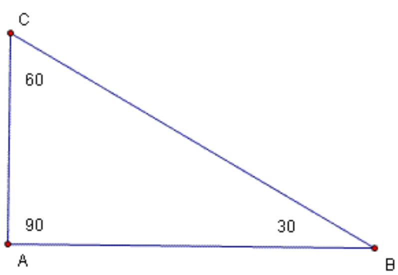


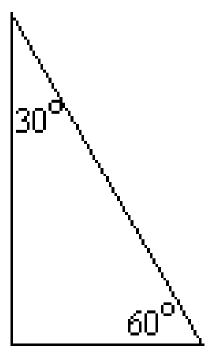
Pythagorean theorem gives $r = \sqrt{x^2 + y^2}$.

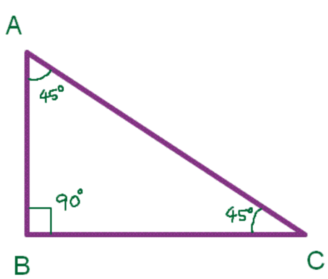
For acute angles, these definitions give the same values as those given by the definitions in Lesson 13.1.

The trigonometric functions are functions only of the angle. Therefore we may choose any radius we please, and the simplest is a circle of radius 1, the Unit Circle.

<u>Fill in:</u> Consider the special right triangles that you already know about, using a hypotenuse of 1.

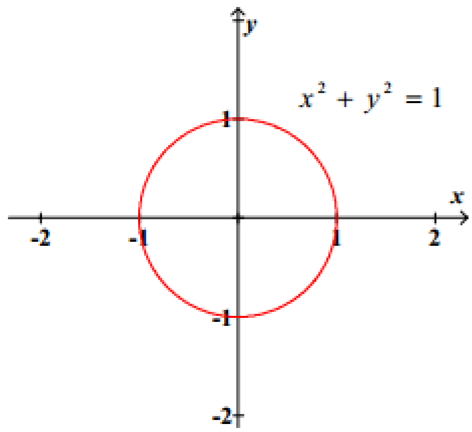






The Unit Circle:

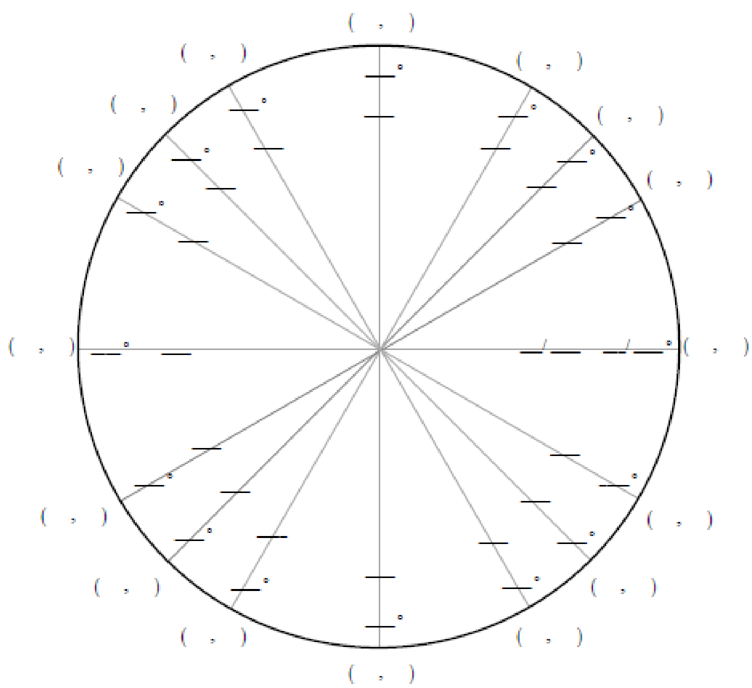
The Unit Circle is the circle centered at the origin with radius 1 unit (hence, the "unit" circle). The equation of this circle is $x^2 + y^2 = 1$. A diagram of the unit circle is shown below:



We have previously applied trigonometry to triangles that were drawn with no reference to any coordinate system. Because the radius of the unit circle is 1, we will see that it provides a convenient framework within which we can apply trigonometry to the coordinate plane.

Now that we are dealing with the Unit Circle we can simplify the trigonometric functions to the ordered pairs (x, y) where x represents $\cos\theta$ and y represents $\sin\theta$ and $\tan\theta = \frac{y}{x}$.

We can now use the unit circle to evaluate the trigonometric values for some of the most frequently used angles.



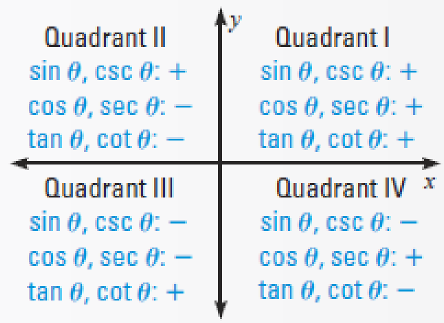
"All Students Take Calculus!"

CONCEPT SUMMARY — **EVALUATING TRIGONOMETRIC FUNCTIONS**

Use these steps to evaluate a trigonometric function of any angle θ.

1. Find the reference angle θ'.
2. Evaluate the trigonometric function for the angle θ'.
3. Use the quadrant in which θ lies to determine the sign of the trigonometric function value of θ. (See the diagram at the right.)

Signs of Function Values



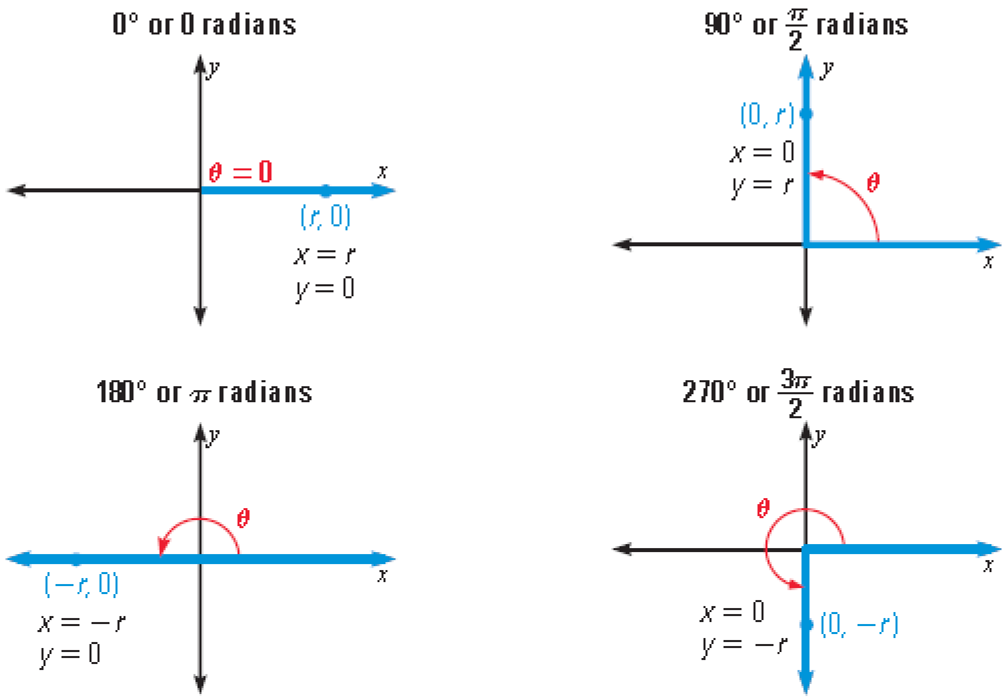
E2. Evaluate the function without using a calculator:

a. $\cos(-150°)$ b. $\cot \frac{16\pi}{3}$ c. $\sin 225°$

P2. Evaluate the function without using a calculator:

a. $\tan(-210°)$ b. $\csc \frac{11\pi}{4}$ c. $\sec \frac{7\pi}{6}$

If the terminal side of θ lies on an axis, then θ is a quadrantal angle. The diagrams below show the values of x and y for the quadrantal angles of 0°, 90°, 180°, and 270°.



E3. Evaluate the six trigonometric functions of θ = 90°.

P3. Evaluate the six trigonometric functions of θ = 180°

Trigonometry Table

Angle					Angle				
Degree	Radian	Sine	Cosine	Tangent	Degree	Radian	Sine	Cosine	Tangent
0°	0.000	0.000	1.000	0.000					
1°	0.017	0.017	1.000	0.017	46°	0.803	0.719	0.695	1.036
2°	0.035	0.035	0.999	0.035	47°	0.820	0.731	0.682	1.072
3°	0.052	0.052	0.999	0.052	48°	0.838	0.743	0.669	1.111
4°	0.070	0.070	0.998	0.070	49°	0.855	0.755	0.656	1.150
5°	0.087	0.087	0.996	0.087	50°	0.873	0.766	0.643	1.192
6°	0.105	0.105	0.995	0.105	51°	0.890	0.777	0.629	1.235
7°	0.122	0.122	0.993	0.123	52°	0.908	0.788	0.616	1.280
8°	0.140	0.139	0.990	0.141	53°	0.925	0.799	0.602	1.327
9°	0.157	0.156	0.988	0.158	54°	0.942	0.809	0.588	1.376
10°	0.175	0.174	0.985	0.176	55°	0.960	0.819	0.574	1.428
11°	0.192	0.191	0.982	0.194	56°	0.977	0.829	0.559	1.483
12°	0.209	0.208	0.978	0.213	57°	0.995	0.839	0.545	1.540
13°	0.227	0.225	0.974	0.231	58°	1.012	0.848	0.530	1.600
14°	0.244	0.242	0.970	0.249	59°	1.030	0.857	0.515	1.664
15°	0.262	0.259	0.966	0.268	60°	1.047	0.866	0.500	1.732
16°	0.279	0.276	0.961	0.287	61°	1.065	0.875	0.485	1.804
17°	0.297	0.292	0.956	0.306	62°	1.082	0.883	0.469	1.881
18°	0.314	0.309	0.951	0.325	63°	1.100	0.891	0.454	1.963
19°	0.332	0.326	0.946	0.344	64°	1.117	0.899	0.438	2.050
20°	0.349	0.342	0.940	0.364	65°	1.134	0.906	0.423	2.145
21°	0.367	0.358	0.934	0.384	66°	1.152	0.914	0.407	2.246
22°	0.384	0.375	0.927	0.404	67°	1.169	0.921	0.391	2.356
23°	0.401	0.391	0.921	0.424	68°	1.187	0.927	0.375	2.475
24°	0.419	0.407	0.914	0.445	69°	1.204	0.934	0.358	2.605
25°	0.436	0.423	0.906	0.466	70°	1.222	0.940	0.342	2.748
26°	0.454	0.438	0.899	0.488	71°	1.239	0.946	0.326	2.904
27°	0.471	0.454	0.891	0.510	72°	1.257	0.951	0.309	3.078
28°	0.489	0.469	0.883	0.532	73°	1.274	0.956	0.292	3.271
29°	0.506	0.485	0.875	0.554	74°	1.292	0.961	0.276	3.487
30°	0.524	0.500	0.866	0.577	75°	1.309	0.966	0.259	3.732
31°	0.541	0.515	0.857	0.601	76°	1.326	0.970	0.242	4.011
32°	0.559	0.530	0.848	0.625	77°	1.344	0.974	0.225	4.332
33°	0.576	0.545	0.839	0.649	78°	1.361	0.978	0.208	4.705
34°	0.593	0.559	0.829	0.675	79°	1.379	0.982	0.191	5.145
35°	0.611	0.574	0.819	0.700	80°	1.396	0.985	0.174	5.671
36°	0.628	0.588	0.809	0.727	81°	1.414	0.988	0.156	6.314
37°	0.646	0.602	0.799	0.754	82°	1.431	0.990	0.139	7.115
38°	0.663	0.616	0.788	0.781	83°	1.449	0.993	0.122	8.144
39°	0.681	0.629	0.777	0.810	84°	1.466	0.995	0.105	9.514
40°	0.698	0.643	0.766	0.839	85°	1.484	0.996	0.087	11.43
41°	0.716	0.656	0.755	0.869	86°	1.501	0.998	0.070	14.30
42°	0.733	0.669	0.743	0.900	87°	1.518	0.999	0.052	19.08
43°	0.750	0.682	0.731	0.933	88°	1.536	0.999	0.035	28.64
44°	0.768	0.695	0.719	0.966	89°	1.553	1.000	0.017	57.29
45°	0.785	0.707	0.707	1.000	90°	1.571	1.000	0.000	

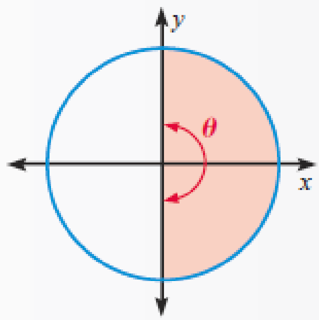
13.4 Inverse Trigonometric Functions (I/2)

So far we have learned how to evaluate trigonometric functions of a given angle. Now we are going to study the reverse, that is, finding angles that correspond to a given value of a trigonometric function.

General definitions of inverse sine, inverse cosine and inverse tangent are given below.

INVERSE TRIGONOMETRIC FUNCTIONS

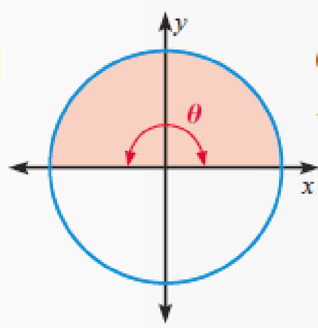
- If $-1 \le a \le 1$, then the **inverse sine** of a is $\sin^{-1} a = \theta$ where $\sin \theta = a$ and $-\frac{\pi}{2} \le \theta \le \frac{\pi}{2}$ (or $-90° \le \theta \le 90°$).



$-\frac{\pi}{2} \le \theta \le \frac{\pi}{2}$

$-1 \le \sin \theta \le 1$

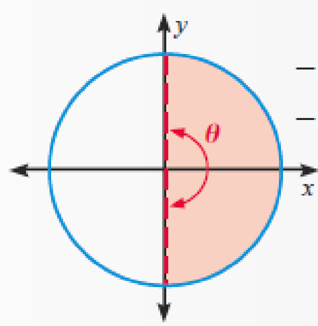
- If $-1 \le a \le 1$, then the **inverse cosine** of a is $\cos^{-1} a = \theta$ where $\cos \theta = a$ and $0 \le \theta \le \pi$ (or $0° \le \theta \le 180°$).



$0 \le \theta \le \pi$

$-1 \le \cos \theta \le 1$

- If a is any real number, then the **inverse tangent** of a is $\tan^{-1} a = \theta$ where $\tan \theta = a$ and $-\frac{\pi}{2} < \theta < \frac{\pi}{2}$ (or $-90° < \theta < 90°$).



$-\frac{\pi}{2} < \theta < \frac{\pi}{2}$

$-\infty < \tan \theta < +\infty$

E1. Evaluate each expression in both radians and degrees without using a calculator.

a. $\sin^{-1}\frac{\sqrt{2}}{2}$ b. $\cos^{-1} 3$ c. $\tan^{-1} 1$

P1. Evaluate each expression in both radians and degrees without using a calculator.

a. $\sin^{-1}\frac{\sqrt{3}}{2}$ b. $\cos^{-1} 2$ c. $\tan^{-1} -1$

E2. Evaluate each expression in both radians and degrees without using a calculator.

a. $\sin^{-1} 0$ b. $\cos^{-1} 0$ c. $\tan^{-1}\frac{\sqrt{3}}{3}$

P2. Evaluate each expression in both radians and degrees without using a calculator.

a. $\sin^{-1}\frac{1}{2}$ b. $\cos^{-1}\frac{1}{2}$ c. $\tan^{-1} 1$

E3. Solve the equation $\cos\theta = -\frac{2}{3}$ where $180° < \theta < 270°$

P3. Solve the equation $\sin\theta = -\frac{1}{4}$ where $180° < \theta < 270°$

E4. Solve the equation $\tan\theta = -5$ where $90° < \theta < 180°$

P4. Solve the equation $\sin\theta = \frac{7}{20}$ where $180° < \theta < 270°$

To solve a triangle with no right angle, you need to know the measure of at least one side and any two other parts of the triangle. This leads to four possible scenarios:

(1) Two angles and any side (AAS or ASA)
(2) Two sides and an angle opposite one of them (SSA)
(3) Three sides (SSS)
(4) Two sides and their included angle (SAS)

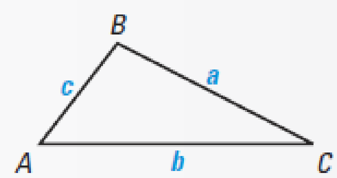
The first two cases can be solved using the Law of Sines. The last two using the Law of Cosines.

LAW OF SINES

If $\triangle ABC$ has sides of length a, b, and c as shown, then:

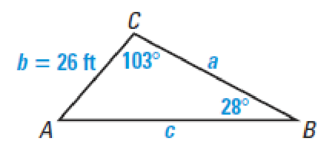
$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

An equivalent form is $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$.



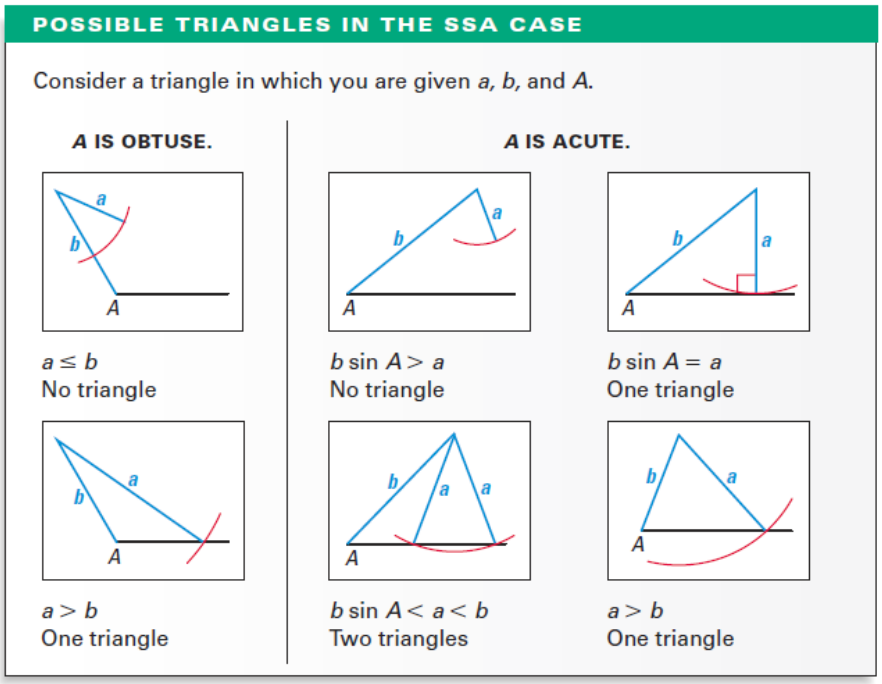
Keep in mind that you only use two ratios at a time to form a proportion equation to solve.

(Ex1.) Solve ΔABC with $m\sphericalangle C = 103°, m\sphericalangle B = 28°, and\ b = 26\ feet.$
Note: This is a case of AAS



(P1.) Solve ΔABC with $m\sphericalangle B = 118°, m\sphericalangle C = 36°, and\ c = 14\ inches.$
Note: This is a case of AAS

The above cases, as you can recall from geometry, determine exactly one triangle. You may also recall that there was no way to use SSA to prove two triangles congruent. This was because when you have SSA you may have no triangle, one triangle or two triangles. This is known as the ambiguous case.



(E2.) Solve ΔABC with $m\sphericalangle C = 122°, a = 12\ cm, and\ c = 18\ cm.$
Note: This is a a case of SSA. This can only form one acute triangle.

(P2.) Solve ΔABC with $m\sphericalangle C = 96°, b = 17\ cm, and\ c = 19\ cm.$
Note: This is a case of SSA. This can only form one acute triangle.

(E3.) Solve ΔABC with $a = 4\ inches, b = 2.5\ inches, and\ m\sphericalangle B = 58°$.
Note: This is a a case of SSA. This forms NO triangle.

(P3.) Solve ΔABC with $c = 8\ inches, b = 11\ inches, and\ m\sphericalangle C = 63°$.
Note: This is a a case of SSA. This forms NO triangle.

(E4.) At certain times during the year, you can see Venus in the morning sky. The distance between Venus and the sun is approximately 67 million miles. The distance between Earth and the sun is approximately 93 million miles. Estimate the distance between Venus and Earth if the observed angle between the sun and Venus is $34°$.

(P4.) The distance from Mercury to the sun is about 36 million miles. The distance from Earth to the sun is about 93 million miles. Estimate the distance from Earth to Mercury if the observed angle between the sun and Mercury is $19°$.

In order to solve these other two cases that we learned about in the last section we need to learn the Law of Cosines:

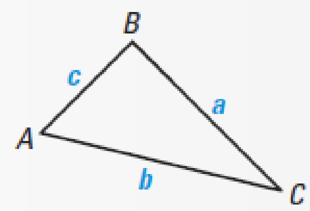
(3) Three sides (SSS)

(4) Two sides and their included angle (SAS)

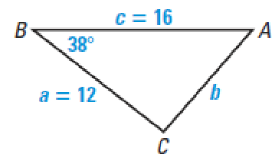
LAW OF COSINES

If $\triangle ABC$ has sides of length a, b, and c as shown, then:

$$a^2 = b^2 + c^2 - 2bc\cos A$$
$$b^2 = a^2 + c^2 - 2ac\cos B$$
$$c^2 = a^2 + b^2 - 2ab\cos C$$

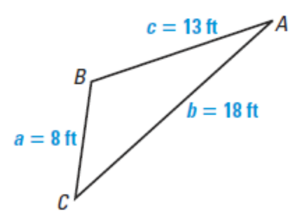


(E1.) Solve $\Delta ABC\ with\ a = 12, c = 16\ and\ m\sphericalangle B = 38°$



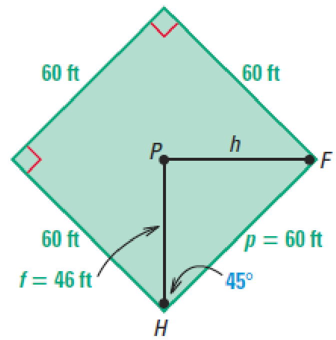
(P1.) Solve $\Delta ABC\ with\ a = 22, c = 19\ and\ m\sphericalangle B = 81°$

(E2.) Solve $\Delta ABC\ with\ a = 8\ feet, b = 18\ feet\ \ and\ c = 13\ feet$



(P2.) Solve $\Delta ABC\ with\ a = 8\ , b = 12\ \ and\ c = 10$

(E3.) The pitcher's mound on a softball field is 46 feet from home plate. The distance between the bases is 60 feet. How far is the pitcher's mound from first base?



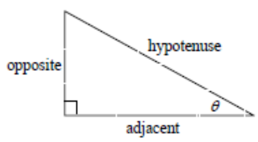
(P3.) In a junior baseball league, the pitcher's mound is 40 feet from home plate, and the bases are 55 feet apart on the square field. How far is the mound from first base?

Trig Cheat Sheet

Definition of the Trig Functions

Right triangle definition
For this definition we assume that

$0<\theta<\frac{\pi}{2}$ or $0°<\theta<90°$.

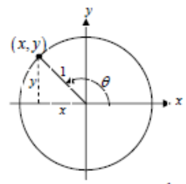


$\sin\theta=\frac{\text{opposite}}{\text{hypotenuse}}$ $\quad\csc\theta=\frac{\text{hypotenuse}}{\text{opposite}}$

$\cos\theta=\frac{\text{adjacent}}{\text{hypotenuse}}$ $\quad\sec\theta=\frac{\text{hypotenuse}}{\text{adjacent}}$

$\tan\theta=\frac{\text{opposite}}{\text{adjacent}}$ $\quad\cot\theta=\frac{\text{adjacent}}{\text{opposite}}$

Unit circle definition
For this definition θ is any angle.



$\sin\theta=\frac{y}{1}=y$ $\quad\csc\theta=\frac{1}{y}$

$\cos\theta=\frac{x}{1}=x$ $\quad\sec\theta=\frac{1}{x}$

$\tan\theta=\frac{y}{x}$ $\quad\cot\theta=\frac{x}{y}$

Facts and Properties

Domain
The domain is all the values of θ that can be plugged into the function.

$\sin\theta$, θ can be any angle
$\cos\theta$, θ can be any angle
$\tan\theta$, $\theta\neq\left(n+\frac{1}{2}\right)\pi,\ n=0,\pm1,\pm2,\ldots$
$\csc\theta$, $\theta\neq n\pi,\ n=0,\pm1,\pm2,\ldots$
$\sec\theta$, $\theta\neq\left(n+\frac{1}{2}\right)\pi,\ n=0,\pm1,\pm2,\ldots$
$\cot\theta$, $\theta\neq n\pi,\ n=0,\pm1,\pm2,\ldots$

Range
The range is all possible values to get out of the function.

$-1\le\sin\theta\le1$ $\quad\csc\theta\ge1$ and $\csc\theta\le-1$
$-1\le\cos\theta\le1$ $\quad\sec\theta\ge1$ and $\sec\theta\le-1$
$-\infty\le\tan\theta\le\infty$ $\quad-\infty\le\cot\theta\le\infty$

Period
The period of a function is the number, T, such that $f(\theta+T)=f(\theta)$. So, if ω is a fixed number and θ is any angle we have the following periods.

$\sin(\omega\theta)\ \rightarrow\ T=\frac{2\pi}{\omega}$
$\cos(\omega\theta)\ \rightarrow\ T=\frac{2\pi}{\omega}$
$\tan(\omega\theta)\ \rightarrow\ T=\frac{\pi}{\omega}$
$\csc(\omega\theta)\ \rightarrow\ T=\frac{2\pi}{\omega}$
$\sec(\omega\theta)\ \rightarrow\ T=\frac{2\pi}{\omega}$
$\cot(\omega\theta)\ \rightarrow\ T=\frac{\pi}{\omega}$

Formulas and Identities

Tangent and Cotangent Identities

$\tan\theta=\frac{\sin\theta}{\cos\theta}$ $\quad\cot\theta=\frac{\cos\theta}{\sin\theta}$

Reciprocal Identities

$\csc\theta=\frac{1}{\sin\theta}$ $\quad\sin\theta=\frac{1}{\csc\theta}$

$\sec\theta=\frac{1}{\cos\theta}$ $\quad\cos\theta=\frac{1}{\sec\theta}$

$\cot\theta=\frac{1}{\tan\theta}$ $\quad\tan\theta=\frac{1}{\cot\theta}$

Pythagorean Identities

$\sin^2\theta+\cos^2\theta=1$
$\tan^2\theta+1=\sec^2\theta$
$1+\cot^2\theta=\csc^2\theta$

Even/Odd Formulas

$\sin(-\theta)=-\sin\theta$ $\quad\csc(-\theta)=-\csc\theta$
$\cos(-\theta)=\cos\theta$ $\quad\sec(-\theta)=\sec\theta$
$\tan(-\theta)=-\tan\theta$ $\quad\cot(-\theta)=-\cot\theta$

Periodic Formulas
If n is an integer.

$\sin(\theta+2\pi n)=\sin\theta$ $\quad\csc(\theta+2\pi n)=\csc\theta$
$\cos(\theta+2\pi n)=\cos\theta$ $\quad\sec(\theta+2\pi n)=\sec\theta$
$\tan(\theta+\pi n)=\tan\theta$ $\quad\cot(\theta+\pi n)=\cot\theta$

Double Angle Formulas

$\sin(2\theta)=2\sin\theta\cos\theta$
$\cos(2\theta)=\cos^2\theta-\sin^2\theta$
$\quad=2\cos^2\theta-1$
$\quad=1-2\sin^2\theta$
$\tan(2\theta)=\frac{2\tan\theta}{1-\tan^2\theta}$

Degrees to Radians Formulas
If x is an angle in degrees and t is an angle in radians then

$\frac{\pi}{180}=\frac{t}{x}\quad\Rightarrow\quad t=\frac{\pi x}{180}\quad$ and $\quad x=\frac{180t}{\pi}$

Half Angle Formulas

$\sin^2\theta=\frac{1}{2}(1-\cos(2\theta))$

$\cos^2\theta=\frac{1}{2}(1+\cos(2\theta))$

$\tan^2\theta=\frac{1-\cos(2\theta)}{1+\cos(2\theta)}$

Sum and Difference Formulas

$\sin(\alpha\pm\beta)=\sin\alpha\cos\beta\pm\cos\alpha\sin\beta$
$\cos(\alpha\pm\beta)=\cos\alpha\cos\beta\mp\sin\alpha\sin\beta$
$\tan(\alpha\pm\beta)=\frac{\tan\alpha\pm\tan\beta}{1\mp\tan\alpha\tan\beta}$

Product to Sum Formulas

$\sin\alpha\sin\beta=\frac{1}{2}\left[\cos(\alpha-\beta)-\cos(\alpha+\beta)\right]$
$\cos\alpha\cos\beta=\frac{1}{2}\left[\cos(\alpha-\beta)+\cos(\alpha+\beta)\right]$
$\sin\alpha\cos\beta=\frac{1}{2}\left[\sin(\alpha+\beta)+\sin(\alpha-\beta)\right]$
$\cos\alpha\sin\beta=\frac{1}{2}\left[\sin(\alpha+\beta)-\sin(\alpha-\beta)\right]$

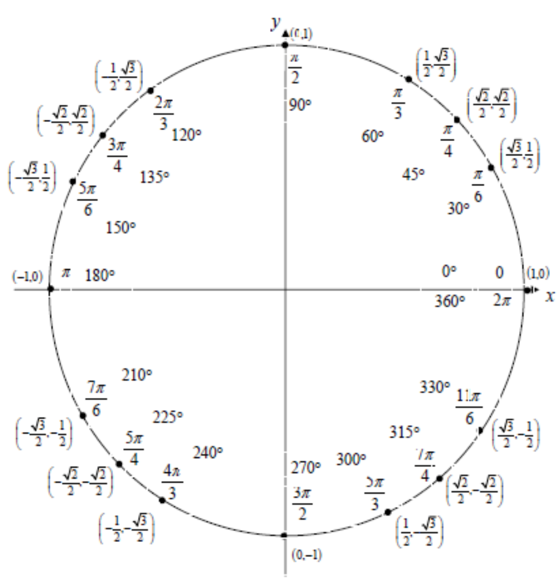
Sum to Product Formulas

$\sin\alpha+\sin\beta=2\sin\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right)$
$\sin\alpha-\sin\beta=2\cos\left(\frac{\alpha+\beta}{2}\right)\sin\left(\frac{\alpha-\beta}{2}\right)$
$\cos\alpha+\cos\beta=2\cos\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right)$
$\cos\alpha-\cos\beta=-2\sin\left(\frac{\alpha+\beta}{2}\right)\sin\left(\frac{\alpha-\beta}{2}\right)$

Cofunction Formulas

$\sin\left(\frac{\pi}{2}-\theta\right)=\cos\theta$ $\quad\cos\left(\frac{\pi}{2}-\theta\right)=\sin\theta$
$\csc\left(\frac{\pi}{2}-\theta\right)=\sec\theta$ $\quad\sec\left(\frac{\pi}{2}-\theta\right)=\csc\theta$
$\tan\left(\frac{\pi}{2}-\theta\right)=\cot\theta$ $\quad\cot\left(\frac{\pi}{2}-\theta\right)=\tan\theta$

Unit Circle



For any ordered pair on the unit circle (x,y): $\cos\theta=x$ and $\sin\theta=y$

Example

$\cos\left(\frac{5\pi}{3}\right)=\frac{1}{2}$ $\qquad\sin\left(\frac{5\pi}{3}\right)=-\frac{\sqrt{3}}{2}$

Inverse Trig Functions

Definition

$y=\sin^{-1}x$ is equivalent to $x=\sin y$
$y=\cos^{-1}x$ is equivalent to $x=\cos y$
$y=\tan^{-1}x$ is equivalent to $x=\tan y$

Domain and Range

Function	Domain	Range
$y=\sin^{-1}x$	$-1\le x\le1$	$-\frac{\pi}{2}\le y\le\frac{\pi}{2}$
$y=\cos^{-1}x$	$-1\le x\le1$	$0\le y\le\pi$
$y=\tan^{-1}x$	$-\infty<x<\infty$	$-\frac{\pi}{2}<y<\frac{\pi}{2}$

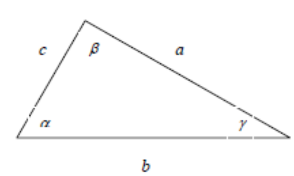
Inverse Properties

$\cos(\cos^{-1}(x))=x$ $\quad\cos^{-1}(\cos(\theta))=\theta$
$\sin(\sin^{-1}(x))=x$ $\quad\sin^{-1}(\sin(\theta))=\theta$
$\tan(\tan^{-1}(x))=x$ $\quad\tan^{-1}(\tan(\theta))=\theta$

Alternate Notation

$\sin^{-1}x=\arcsin x$
$\cos^{-1}x=\arccos x$
$\tan^{-1}x=\arctan x$

Law of Sines, Cosines and Tangents



Law of Sines

$\frac{\sin\alpha}{a}=\frac{\sin\beta}{b}=\frac{\sin\gamma}{c}$

Law of Cosines

$a^2=b^2+c^2-2bc\cos\alpha$
$b^2=a^2+c^2-2ac\cos\beta$
$c^2=a^2+b^2-2ab\cos\gamma$

Mollweide's Formula

$\frac{a+b}{c}=\frac{\cos\frac{1}{2}(\alpha-\beta)}{\sin\frac{1}{2}\gamma}$

Law of Tangents

$\frac{a-b}{a+b}=\frac{\tan\frac{1}{2}(\alpha-\beta)}{\tan\frac{1}{2}(\alpha+\beta)}$

$\frac{b-c}{b+c}=\frac{\tan\frac{1}{2}(\beta-\gamma)}{\tan\frac{1}{2}(\beta+\gamma)}$

$\frac{a-c}{a+c}=\frac{\tan\frac{1}{2}(\alpha-\gamma)}{\tan\frac{1}{2}(\alpha+\gamma)}$

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